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REDUCTION OF FALSE ALARM RATES IN PERIPHERAL DETECTION SYSTEMS

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1 February 1980

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The distribution function of the time between false alarms which results from combining the output of two independent sensors in an AND-logic mode is derived. In particular, the false alarms from each sensor are assumed to be distributed according to Poisson laws (λ_1 and λ_2). A "moving window" with a time interval τ is used such that an alarm for the combined system is triggered if, and only if, an alarm from each system occurs within an interval τ. Numerical results of using the derived distribution in combination with a simplified detection model are presented for a number of cases.

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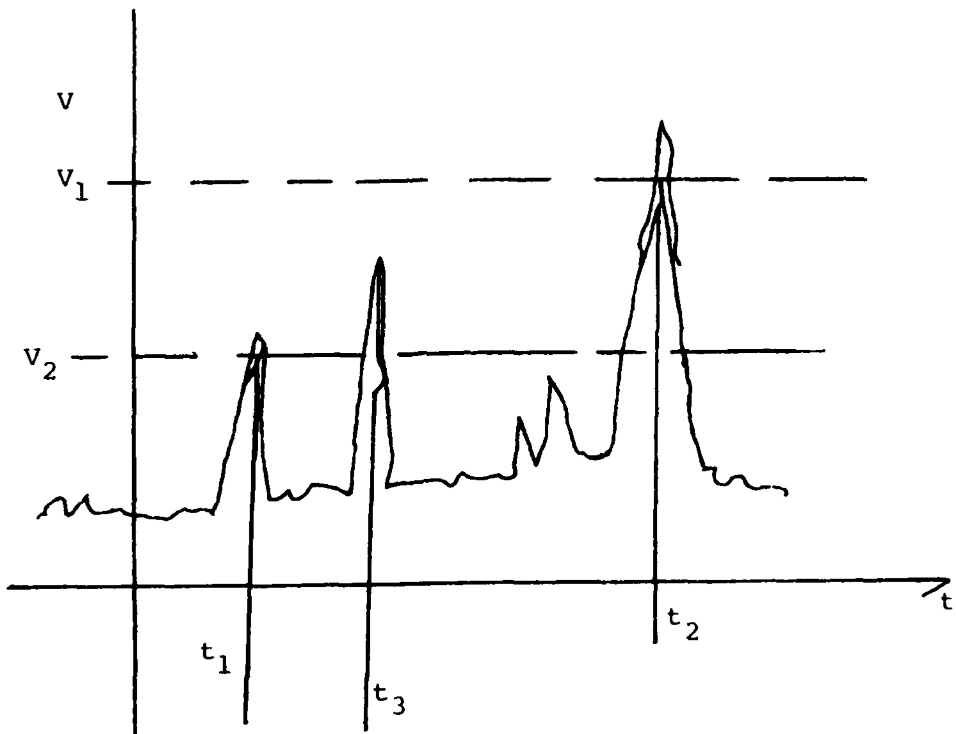
INTRODUCTION

Present day peripheral detection systems exhibit high probabilities of intruder detection. Unfortunately, this improved detection performance may often be accompanied by a significant increase in false alarm rate. For an installation with a number of these devices deployed on its periphery, the total rate produced during certain periods of operation can impose a severe burden on the alarm processing function as well as result in degraded response force performance. The purpose of this discussion is to examine analytically an approach to the false alarm problem which does not involve extensive modification of current devices and systems. This approach consists of combining the output of two sensors using AND-logic.

SYSTEM CHARACTERISTICS OF PERIPHERAL SENSORS

The typical detection device produces a voltage signal as a function of time. Alarms are triggered when the voltage exceeds a given threshold which is chosen to produce the desired probability of detection. Figure 1 shows a possible time history produced by the sensor. Signals produced by an intrusion are indicated at times t_1 and t_2. If the threshold is set at V_1, the decision logic will trigger an alarm at time t_2; but, the signal at time t_1 will fail to produce an alarm. If the threshold is set at V_2, both intrusion signals will produce alarms; but, the peak shown at time t_3 which is the result of noise in the system, will also produce an alarm. Thus, lowering the threshold leads to more valid alarms and also increases the probability of false alarms.



Signals Resulting from Intrusions are at Times t_1 & t_2.
Peak at Time t_3 is the Result of "Noise" in the System.

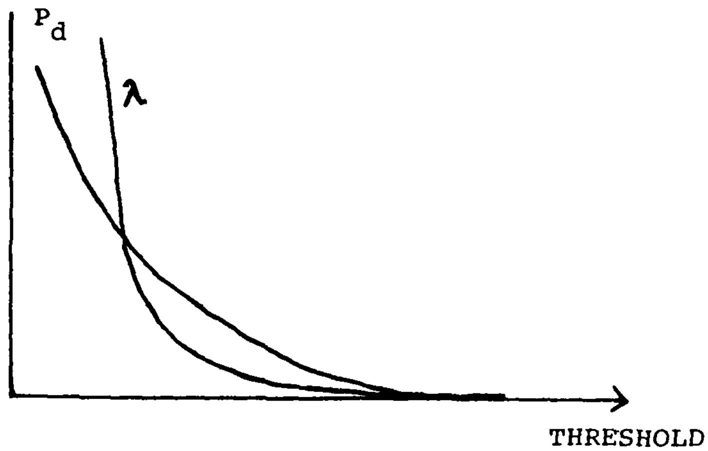
Figure 1. TYPICAL OUTPUT OF SENSING DEVICE - VOLTAGE VERSUS TIME

In general, false alarm rate, λ, and detection probability, P_d, are related quantities: increasing the probability of detection generally leads to higher false alarm rates and lowering false alarm rates leads to decreased probability of detection. Figure 2 shows the typical relationship between the detection probability and false alarm rates as a function of threshold setting.

The detection performance of individual sensors is affected by a number of factors. Sensors will perform differently, in different environments. Buried sensors will have different responses in frozen ground, snow cover and various soil conditions. Above ground sensors will be affected by rain, fog and other effects. Intruders will exhibit different sizes, knowledge and modes of approach, and they will move at different velocities (this is a key factor in combined system performance). Finally, P_d is a function of range from the sensor.

Even more important are the individual sensor characteristics. Sensors can emit one or more alarms per valid intrusion. There may be variable time delays introduced as a result of the uncertainties in predicting classification times and communication delays. Sensors may exhibit a blanking interval (or lockout time) after the first alarm. Deployment variables may vary greatly. Individual sensors may be installed above ground or buried; they may be wide area, line or point types and they may have coverage patterns which are time variant (e.g., radars). Sensors in combination may be installed such that they are coincident, provide overlapping coverage, or in a sequential array. The P_d versus range statistics may vary greatly by sensor type (e.g., $1/R^3$, $1/R^4$, exponential, gaussian, etc.). Finally, false alarms may exhibit different arrival time statistics which are also dependent upon environmental variations. These must all be considered in configuring a combination system.

False alarms arise from natural disturbances (rain, wind, thunder, etc.), cultural background "noise" (industrial noise,



P_d = Probability of Detection, Given an Intrusion

λ = False Alarm Rate

Figure 2. TYPICAL RELATIONSHIP BETWEEN P_d AND λ AS FUNCTIONS OF THRESHOLD SETTING.

vehicular traffic) and internally generated noise. The alarms from two different sensors may also be highly correlated. Thus, two fence sensors in the same vicinity will be affected by the same gusts of wind. Thunder could similarly trigger correlated alarms in acoustic sensors. In addition, nuisance alarms can be produced by the wild-life inhabiting certain areas.

It should also be pointed out that the false alarm rate at a given location is not a constant through all time. There may be significant diurnal variations due to changes in the environment such as the ebb and flow of vehicular traffic or industrial activity in the area. The occurrence of storms is also a factor in this variation.

Table 1 is a summary of the detection and false alarm characteristics of the devices under consideration.

FALSE ALARM REDUCTION

Several approaches are possible for the false alarm reduction problem. One of these is to install a more sophisticated decision process in the logic of the detection device. As stated above, the current alarm/no alarm decision procedure is based on simple thresholding. False alarms arising from specific disturbances such as thunder could, in principle, be filtered out of the system by processing the waveforms associated with them. That is, the characteristic waveforms associated with various kinds of disturbances could be used in a cross-correlation (or other pattern recognition) mode to permit classification of the source of the disturbance. Such an approach involves a systematic examination of the ensemble of waveforms associated with the phenomena in order to generate the algorithm or logic necessary for the decision process. Referring to Figure 1, it would be necessary to isolate the cause of the peal at time t_3 and to examine the time history of that transient in the system. By analyzing the characteristics of this ensemble a filter could be constructed which allows identification of a particular set of

Table 1. SUMMARY OF DETECTION AND FALSE ALARM CHARACTERISTICS

Variables Affecting P_d	Variables Affecting False Alarm Rate	Sensor Characteristics
• Environment	• Environment	• Multiple Alarms
• Type of Intruder	• Cultural	• Variable Time Delays
• Velocity of Intruder	• System Noise	• Lockout Time
• Range		• Development Configurations
		• Detection Range Distribution
		• False Alarm Distribution

disturbances. The success of such an approach is problematical in that sufficient data have not been scrutinized at this point to establish the degree of uniqueness of the waveforms associated with actual intrusions. Thus, it is not clear to what extent a transient due to wind acting on a fence sensor is different from that produced by an intruder. In any case, the performance of such an analysis would require extensive data collection of the waveforms and their associated causes. At present, the decision process resides in the device that yields the alarm/no alarm decision. Thus, the information available is of a binary go/no-go nature since the decision is made by thresholding.

Another possible approach is the implementation of a stochastic threshold $V_o(t)$ which is a function of time and background conditions. The classical Wiener spectrum method does not apply here since the statistics of the background noise are not stationary. Much work has been done in recent years, however, which applies to adaptive thresholding in the presence of non-stationary noise (References 1 through 3).

A third approach is to use two or more devices in an area tracking mode. By logically processing the times and locations of the alarms, an attempt could be made to establish a track associated with a given intrusion. Failure to establish a track would indicate that no intrusion had, in fact, taken place and that certain of the alarms were spurious.

A fourth approach to false alarm reduction and the one examined in detail in this report is to use two devices in a modified tracking mode. This approach has the advantage of not requiring modification of existing detection devices. It does not involve changing the threshold method for alarm decisions. Rather, it would process the output of two devices and perform a decision function <u>exterior</u> to both devices.

The case considered will be that of two coincident generically different sensors. The alarm output of each device is monitored and the time between an alarm from system S_i and the first subsequent alarm from system S_j ($i \neq j$) is computed. If that time is greater than a threshold τ, the system does not react. If the interval between alarms from each system is less than τ, the system reacts and an alarm is triggered. The parameter τ is chosen based on the detection characteristics of S_1 and S_2. The rationale for choosing τ will be discussed later in the report. The reason for deploying generically different sensors is that false alarms are less likely to be correlated in this case. In summary, the decision logic consists of deciding a system alarm if, and only if, at least one alarm occurs from each sensor during an interval τ. In the remainder of this report, the analytics of the system logic and numerical examples will be presented. Of particular interest will be a comparison of the average time between alarms for the combined system and that for the systems S_1, and S_2.

Another quantity of interest is the time between alarms that corresponds to, say, the 95-percentile level. That is, the time between alarms such that 95% of the intervals are greater than it.

ANALYTICAL MODEL

The first topic that will be discussed here is the derivation of the probability density function of the time t between system alarms for the case of coincident sensors. To achieve a model which is relatively simple, the following assumptions will be made. First it will be assumed that there are no delays in communicating alarms to the decision processor. Further, there is no back-out time after a detection. The false alarms in S_1 are assumed to obey a Poisson distribution with rate λ_1, and those in S_2 with rate λ_2. The derivation of the density function of t_1 the time to false alarm in the combined system, will proceed on a step-by-step basis. Consider, first, the situation when the system is turned on at time zero. An interval of time t_1 elapses before an alarm from S_1 appears and an interval t_2, before an alarm from S_2. The joint probability density of t_1 and t_2 is:

$$j(t_1,t_2) = \lambda_1 \lambda_2 e^{-\lambda_1 t_1} e^{-\lambda_2 t_2}.$$

At this point there are two possibilities: $t_1 < t_2$ or $t_1 > t_2$. With AND-logic in operation, the combined system will be alarmed if $t_2 - t_1 < \tau$ and $t_2 > t_1$;

or if, $t_2 < t_1$ and $t_1 - t_2 < \tau$.

To obtain the density function of the time t to system alarm, the function $j(t_1, t_2)$ must be integrated over the appropriate interval. For the case $t_1 < t_2$, Figure 3 shows the region of integration associated with the density function. The interval $I(t_2)$ over which $j(t_1,t_2)$ must be integrated is indicated by arrows. Notice that the limits of the interval depend on whether t_2 is less than or greater than τ. When $t_2 < \tau$, the interval is $0 < t_1 < t_2$ and when $t_2 > \tau$, the interval is $t_2 - \tau < t_1 < t_2$. The same arguement holds, mutatis mutandis, for the case where $t_2 < t_1$. Performing the integrations yields the density function of waiting time, given that the event occurs on the first pass as:

$$h_o(t) = \begin{cases} \lambda_1 \left[1-e^{-\lambda_2 t}\right] + \lambda_2 \left[1-e^{-\lambda_1 t}\right], & t \leq \tau \\ e^{-\lambda t}\left[\lambda_1 (e^{\lambda_2 \tau} - 1) + \lambda_2 (e^{\lambda_1 \tau} - 1)\right], & t > \tau; \end{cases}$$

where $\lambda = \lambda_1 + \lambda_2$.

Suppose, now, that the difference $|t_2 - t_1|$ fails to satisfy the criterion, i.e., $|t_2 - t_1| > \tau$ for the first two alarms from both systems S_1 and S_2. For the case $t_2 > t_1$, the interval over which $j(t_1,t_2)$ must be integrated is $t_2 > t_1 + \tau$. Thus, the density function of t is:

$$q_o(t) = e^{-\lambda t}\left|\lambda_1 e^{-\lambda_2 \tau} + \lambda_2 e^{-\lambda_1 \tau}\right|;$$

where both cases ($t_1 < t_2$ and $t_1 > t_2$) have been included in the formula.

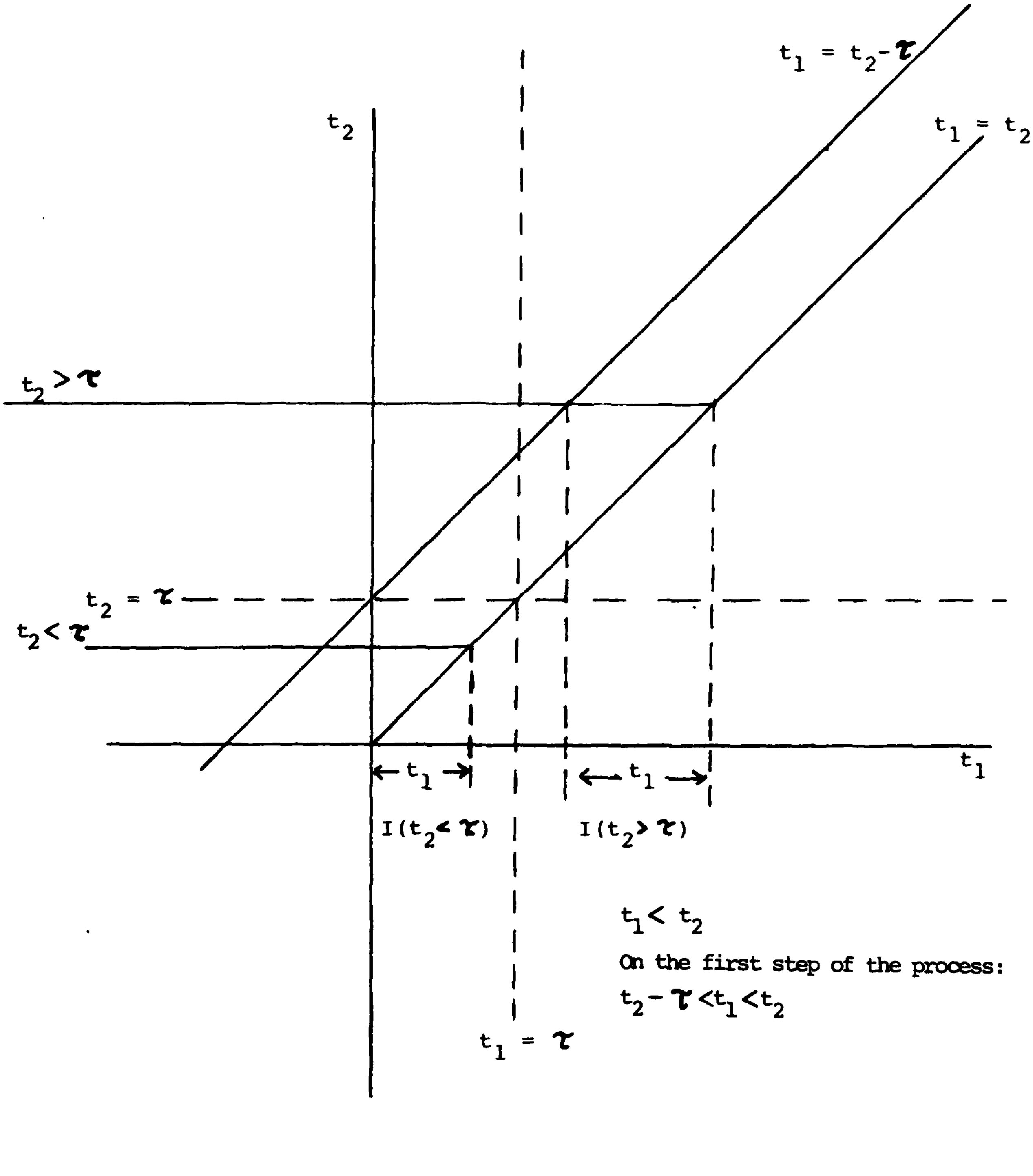


Figure 3. REGION OF INTEGRATION

The above analysis has shown the derivation of the start of the process. Each step of the succeeding process is essentially a repetition of a similar procedure. The point here is that the sequence may be segmented into intervals such that the analysis for each segment is identical. The distribution of waiting time is, then, the convolution of these intervals.

Reference to Figure 4 shows a schematic of the relation of times of alarms from S_1 and S_2. Figure 4 shows the case where the criterion has not been met and the last system to fail is S_1. Figure 4a illustrates the case were the time of the next alarm from S_1 is greater than that from S_2. The marginal density of t_2 for the case when the criterion is met is:

$$g(t_2) = \lambda_2 e^{-\lambda t_2} e^{-\lambda_1 \tau} (1-e^{-\lambda_1 \tau}).$$

The marginal density in the case of failure to meet the criterion is:

$$g^*(t_2) = \lambda_2 e^{-\lambda t_2} e^{-2\tau \lambda_1}.$$

Similarly, Figure 4b illustrates the case when the next alarm from S_1 occurs sooner than that from S_2. The expressions for $g(t_1)$ and $g^*(t_1)$ are similar. Note that when the criterion fails as in Figure 4a, the interval τ must be added to the accrued waiting time, and, when the criterion is met as in Figure 4b, the quantity τ must be added. Otherwise, the total waiting time is the sum of exponential variates with parameter $\lambda = \lambda_1 + \lambda_2$.

When the analysis is carried to completion, the calculation of the density function of waiting time may be formulated as follows. The density function of the sum of j exponential variates is given by:

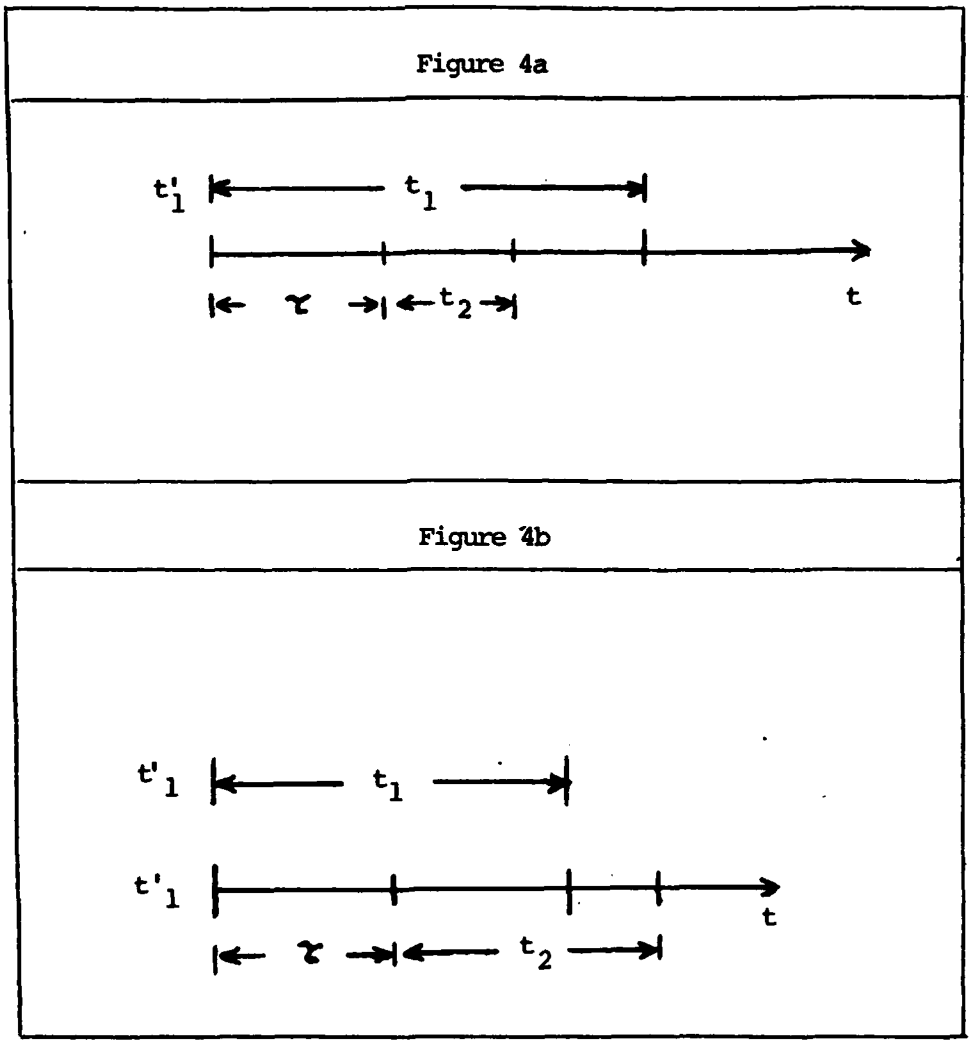


Figure 4. DETERMINATION OF LIMITS OF INTEGRATION

$$g(t',j) = \frac{\lambda^j t'^{(j-1)} e^{-\lambda t'}}{(j-1)!};$$

where t' is the sum and λ is the rate.

The expression for the density function of the waiting time is simplified if a set of auxillary parameters is defined:

$$\lambda = \lambda_1 + \lambda_2; \; p_1 = \frac{\lambda_1}{\lambda}; \; p_2 = \frac{\lambda_2}{\lambda};$$

$$G_1 = p_1 p_2 e^{-2\tau\lambda}$$

$$G_2 = p_1 p_2 \left[p_1 e^{-(\lambda_2\tau + 2\tau\lambda_1)} (1-e^{-2\lambda_2\tau}) + p_2(1-e^{-2\lambda_1\tau}) e^{-(\lambda_1\tau + 2\tau\lambda_2)} \right]$$

$$H_1 = p_1 p_2 e^{-\lambda_2\tau} (1-e^{-2\lambda_1\tau})$$

$$H_2 = p_1 p_2 e^{-\lambda_1\tau} (1-e^{-2\lambda_2\tau}).$$

The functions that explicitly involve t are then written as:

$$R_1(t,1) = \sum_{n=0}^{\infty} g(t-\tau, n+2) p_1^n;$$

$$R_2(t,1) = \sum_{n=0}^{\infty} g(t-\tau, n+2) p_2^n.$$

For $k > 1$,

$$R_o(t,k) = \sum_{n=0}^{\infty} g(t-k\tau, n+k+1) \sum_{j=0}^{n} \binom{j+\frac{k}{2}-1}{j} \binom{n-j+\frac{k}{2}-1}{n-j} p_1^j p_2^{n-j}$$

and

$$R_1(t,k) = \sum_{n=0}^{\infty} g(t-k\tau, n+k+1) \sum_{j=0}^{n} \binom{j+\frac{k+1}{2}-1}{j} \binom{n-j+\frac{k-1}{2}-1}{n-j} p_1^j p_2^{n-j}.$$

The partial densities are then defined as:

$$h(t,k) = \begin{cases} G_1^{\frac{k}{2}-1} G_2 R_o(t,k); \ k \text{ even} \\ G_1^{\frac{k-1}{2}} \left[H_1 R_1(t,k) + H_2 R_2(t,k) \right]; \ k \text{ odd.} \end{cases}$$

Finally, the density function of waiting time can be written as:

$$h(t) = h_o(t) + \sum_{k=1}^{\left[\frac{t}{\tau}\right]} h(t,k);$$

where $\left[x\right]$ is the greatest integer not exceeding x.

The function h(t) can be used to calculate the average or expected waiting time. This is given by:

$$E(t) = \int_0^{\infty} t \, h(t) \, dt.$$

Figure 5 is a plot of E(t) versus λ, for the special case of $\lambda_1 = \lambda_2$ and for a variety of τ's. Also shown on the same plot is the expected waiting time when either system S_1 or S_2 is used alone.

Figure 6 is a plot of the cumulative distribution function H(t) when λ_1=2/hr, λ_2 = 10/hr, and τ= 0.5 min. From this curve it is possible to determine the probability that the waiting time will be less than a given value. For example, if the minimum allowable time, due to operational and other constraints, is 0.2 hr, the curve shows that, for this combination of λ_1, λ_2 and τ, the probability is 0.05 that the waiting time will be less than this.

Thus far, the analysis has dealt with the effect of the use of the "moving window" τ on the time between alarms. The subject of the resultant probability of detection must now be addressed.

Associated with system S_i is a conditional probability P_{Di} which is the probability that an alarm is triggered, given that an intruder

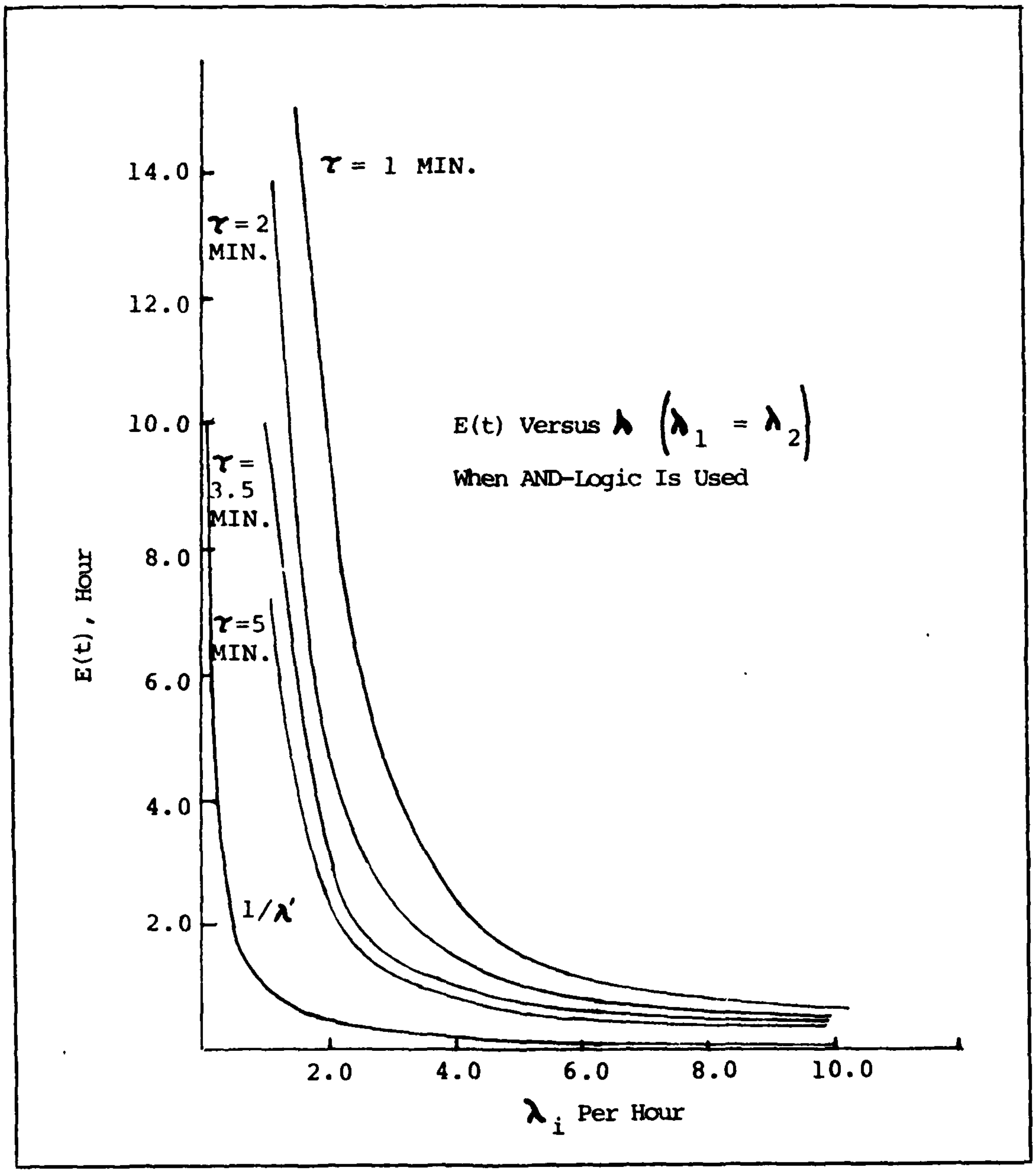


Figure 5. CURVES OF CONSTANT τ in E(t) VERSUS λ

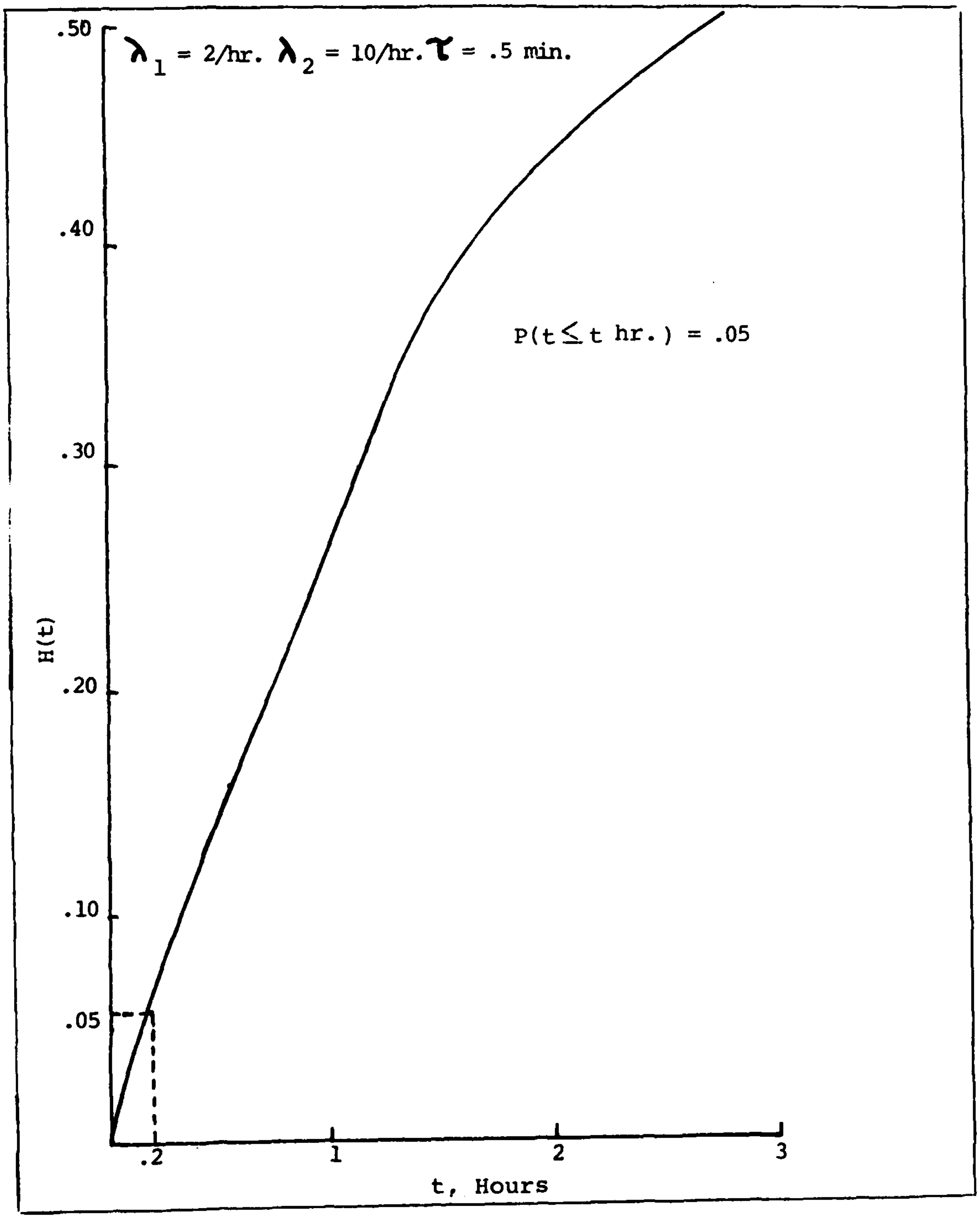


Figure 6. CUMULATIVE DISTRIBUTION FUNCTION OF TIME BETWEEN ALARMS

has passed through the detection zone of that system. The output of the two systems, S_1 and S_2, can be combined to improve the net probability of detection or to lower the false alarm rate. For example, the decision procedure which declares an alarm when either system gives an alarm improves the probability of detection. For this procedure (commonly called OR-logic) the resulting probability of detection is: $P_D = 1-(1-P_{D1})(1-P_{D2})$. The false alarm rate increases, however, and is given by $\lambda = \lambda_1 + \lambda_2$ for the Poisson distribution false alarms discussed above. The AND-logic under consideration requires that both S_1 and S_2 trigger an alarm in an interval τ during the passage of an intruder through the zone of detection. This requirement can be met if S_1 and S_2 give valid alarms due to intruder passage within an interval τ; or if either system gives an alarm which is preceded or followed by a false alarm from the other system. Let γ be the probability that both systems give valid alarms within an interval τ. Then the combined system probability of detection is:

$$P_D = \gamma + P_{D1}(1-e^{-2\lambda_2\tau}) + P_{D2}(1-e^{-2\lambda_1\tau}) + \epsilon .$$

The quantity ϵ appearing in the expression corresponds to the probability that both S_1 and S_2 give a false alarm within an interval τ during intruder passage time. To assess ϵ for any particular time of passage (a function of intruder average velocity and the width of the zone) the cumulative distribution function H(t) may be consulted. In general, the term ϵ may be neglected and the resulting expression for P_D provides a convenient lower bound for the probability of detection.

In this connection it should be observed that, except in the case of very high false alarm rates, the use of AND-logic must lead to a lower combined system probability of detection than that which obtains when either system is used alone. Thus, the upper bond on γ is:

$$\gamma < P_{D1} P_{D2}.$$

This is the price that must be paid for a reduction in false alarm rate.

The crucial term entering into the calculation of P_D is γ, the probability that both systems trigger a valid alarm within an interval τ. To evaluate γ, the characteristics of the sensing device as well as the relevant conditions of the intrusion must be used. Thus, most sensing devices are range-dependent, which means that they are more likely to trigger an alarm as the intruder comes nearer a subzone of the detection region. Also, some devices will have responses that are functions of intruder velocity. In any case, the time that the intruder dwells in the detection zone is an important factor.

To give substance to the above remarks, a simplified, but illuminating sensor model will be introduced. This will permit the calculation of numerical results which will illustrate some of the trade-offs between probability of detection and false alarm rate.

To characterize the performance of the sensors, define $g_1(s_1)$ to be the density function of the probability of detection when the intruder has progressed to a point which is a distance s_1 from Sensor 1 and let $g_2(s_2)$ be the corresponding density function for Sensor 2. Here, it is assumed that there is only one alarm per sensor during an intrusion.

Then $\displaystyle\int_{-\infty}^{\infty} g_i(s_i)\,ds_i = P_{ai}, \quad i = 1.2.$

The critical variable in selecting the time window is $s = |s_2 - s_1|$ or the distance traveled by the intruder between detection by each sensor. For the sake of concreteness, let

$$g_i(s_i) = k_i \exp \frac{-s_i^2}{2\sigma_i^2};$$

where k_1 and k_2 are normalizing factors. Then the density of $s_3 = s_2 - s_1$ is

$$g_3(s_3) = k_3 \exp \frac{-s_3^2}{2\sigma^2};$$

where $\sigma^2 = \sigma_1^2 + \sigma_2^2$; $k_3 = \dfrac{P_{d_1} P_{d_2}}{\sqrt{2\pi}\sqrt{\sigma_1^2 + \sigma_2^2}}$.

The probability that $s < s_o$ is:

$$P(s < s_o) = 2k_3 \int_o^{s_o} \exp \frac{-s^2}{2\sigma_3^2}\, ds;$$

and, hence, the density function of $s = |s_2 - s_1|$ is:

$$f(s) = 2k_3 \exp \frac{-s^2}{2\sigma_1^2}; \quad 0 < s < \infty.$$

The operator of the detection system does not observe s̲ directly, but rather, t̲, the time between detections. Let the average speed of the intruder between detections equal v. The density function of t is given by:

$$f(t) = 2k_3\, v \exp \frac{-v^2 t^2}{2\sigma_3^2}.$$

Thus, it becomes apparent that the density function of t is conditional upon the speed of the intruder as he passes through the zone of detection. The time window, τ, must be chosen so that:

$$2k_3 v \int_o^{\tau} \exp \frac{-v^2 t^2}{2\sigma_3^2}\, dt = \gamma;$$

where γ is an acceptably high probability of detection ($\gamma \leq P_{d1} P_{d2}$).

The parameter v may be treated in a variety of ways. A conservative approach is to set v equal to the minimum velocity that poses a threat when the functioning of the entire system (that in which this detection subsystem is imbedded) is considered. Another approach is to assume a distribution on v and to choose τ based on the expected value of γ. That is, let p(v) be the density function of v, then:

$$E(\gamma) = 2k_3 \int v\, p(v) \int_0^{\tau} \exp \frac{-v^2 t^2}{2\sigma_3^2}\, dt\, dv.$$

For example, assume that v is uniformly distributed between v_o and v_1, then

$$E(\gamma) = \frac{2k_3}{(v_1 - v_o)} \int_{v_o}^{v_1} v dv \int_0^{\tau} \exp \frac{-v^2 t^2}{2\sigma_3^2}\, dt.$$

The total probability of detection for the combined case is

$$P_d = E(\gamma) + P_d(1-P_{d_2})(1-e^{-2\lambda_2 \tau}) + P_{d_2}(1-P_{d_1})(1-e^{-2\lambda_1 \tau}).$$

It has been assumed in the above derivation that each sensor gives a single alarm. This was, of course, implicit in generating the density functions $g_i(s_i)$. In reality, a sensor could give a number of alarms during a given intrusion. Thus, the density g(s) is not adequate for the most general situation. The following discussion will seek to elucidate this point.

It was assumed above that detection depended only on the distance of the intruder from a given zero reference line. As a consequence of this dependence, it was found that the width, τ, of the time window was a function of the speed of the intruder. In general, however, the density function itself may also be a function of intruder speed. Moreover, multiple alarms can occur for a given sensor and intrusion.

Figure 7a is a plot of P_d versus λ for a variety of τ's when $2\sigma_i = 0.7$ meters and $P_{di} = 0.9$. The $2\sigma_i$ value corresponds to a range that accounts for approximately 95% of the detections of either system. Figures 7b and 7c are similar plots for $2\sigma_i = 0.564$ meters and $2\sigma_i = 1.6$ meters, respectively.

Figure 8a is a plot of P_d versus λ for a variety of σ_i's with $\tau = 1$ sec. Figures 8b and 8c are the corresponding plots with $\tau = 2$ seconds and $\tau = 5$ seconds, respectively.

With the analysis thus far completed, it is possible to perform trade-offs between the degradation of the combined system P_d and the reduced false alarm rate. Figure 9a displays such a trade-off for $\lambda = 1/\text{min.}$ and $2\sigma_i = 0.564$ meters. Both P_d and E(t) (expected time to false alarm) are plotted versus τ. Figure 9b is a similar plot for $\lambda = 2/\text{min.}$

As was mentioned above, another figure of merit that may be used to assess the effect of combining two systems in an AND-logic mode is the waiting time corresponding to the 95 percentile. That is the time, t_{min}, such that the probability that the waiting time is greater than t_{min} is 0.95. The parameter t_{min} becomes particularly significant in the case of an installation consisting of numerous zones of detection. There, the parameter t_{min} has an important bearing on the assessment time between alarms. If there are n statistically independent zones, then the distribution of the time between alarms is given by

$$G(t) = 1-(1-H(t))^n;$$

where H(t) is the cumulative distribution function of the time between alarms for a single zone. Figure 10 is a plot of t_{min} versus n for $\lambda_1 = 0.5/\text{min.}$, $\lambda_2 = 2/\text{min}$ and $\tau = 0.3$ sec.

To establish a trade-off for the case of n zones, it is necessary to synthesize the results concerning t_{min} (Figure 10) and those concerning the probability of detection using the

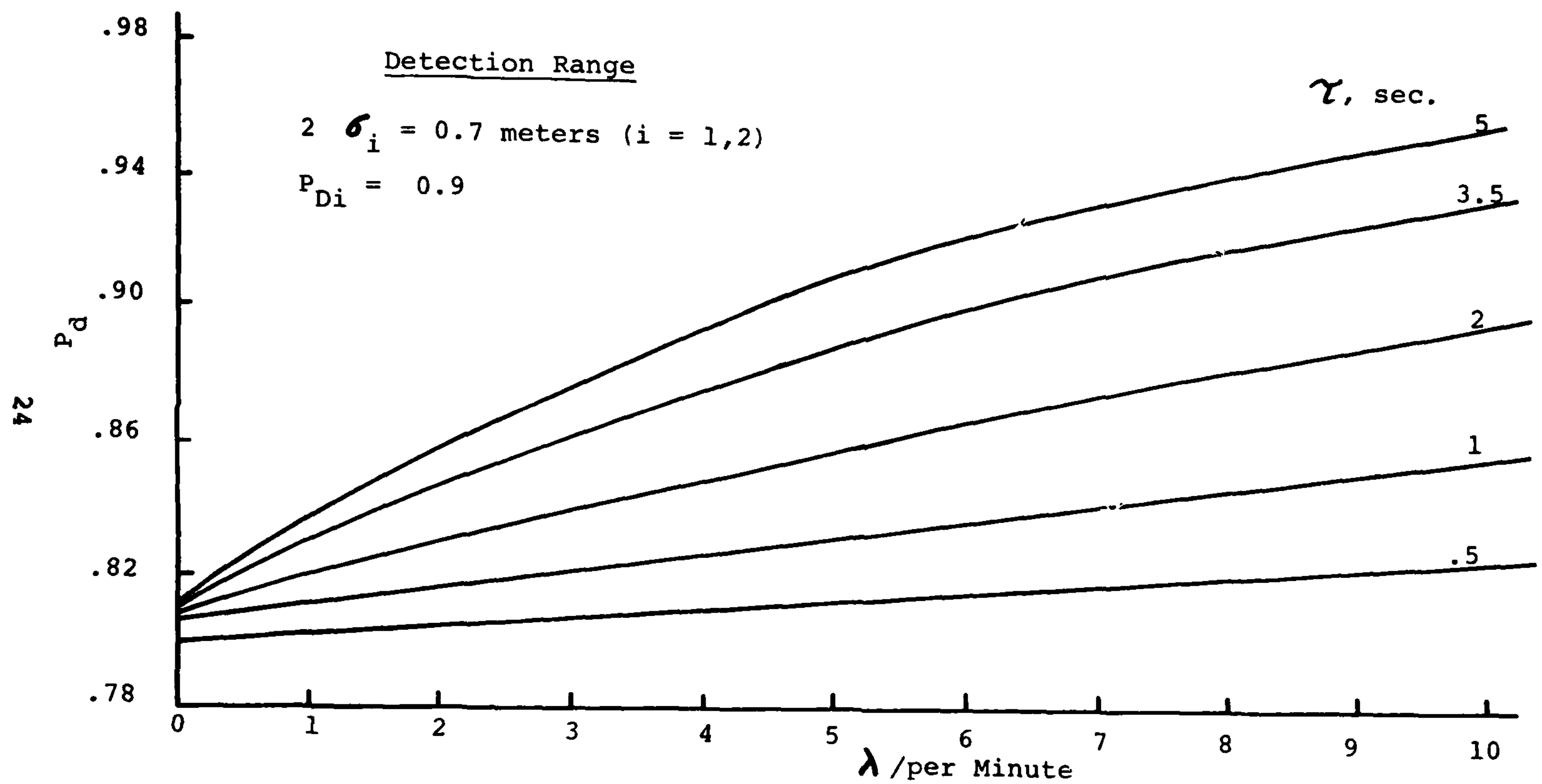


Figure 7a. PROBABILITY OF DETECTION FOR SENSOR SYSTEM (2 σ_i = 0.7m)

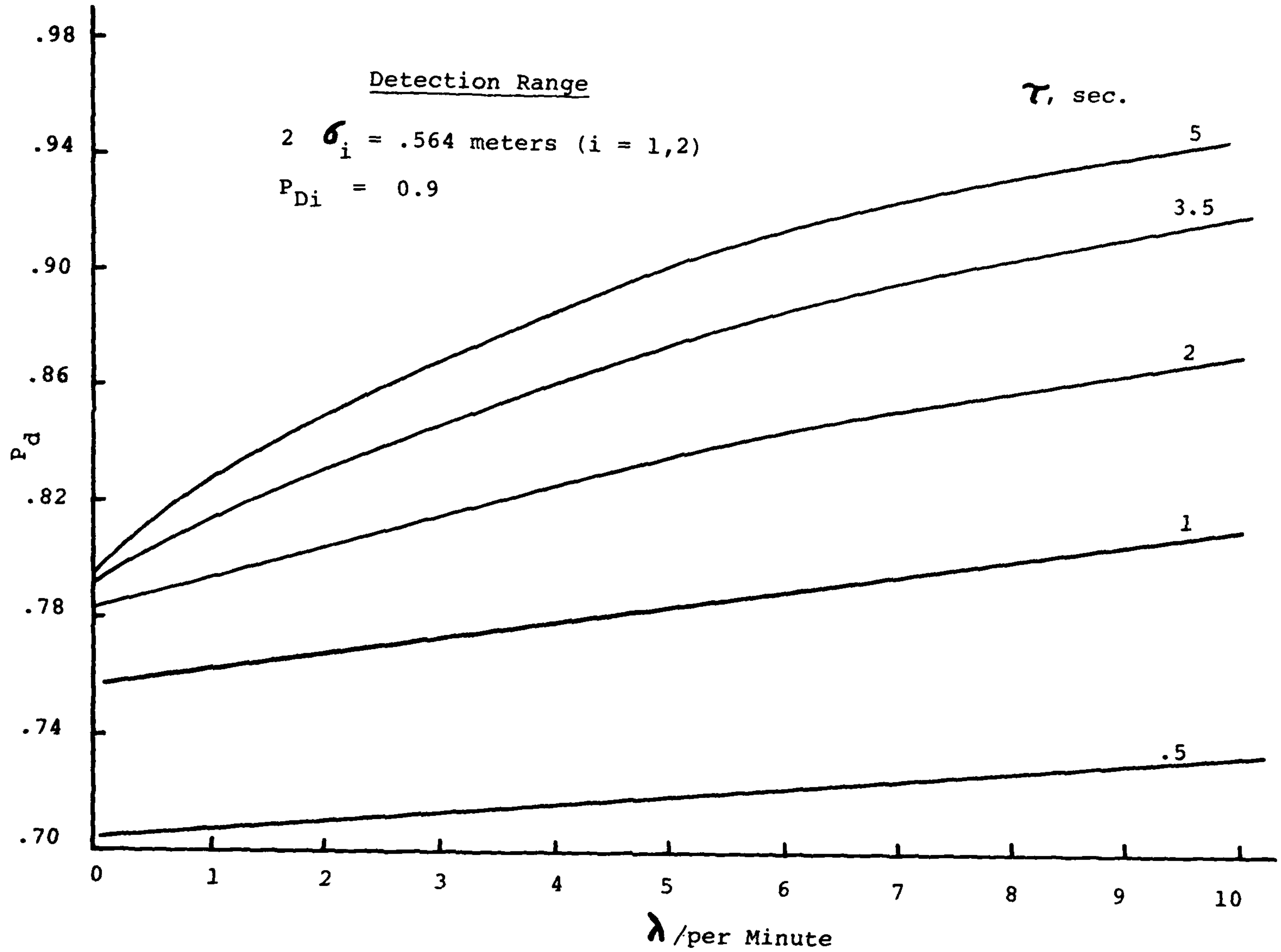


Figure 7b. PROBABILITY OF DETECTION FOR SENSOR SYSTEM (2 σ_i = .564m)

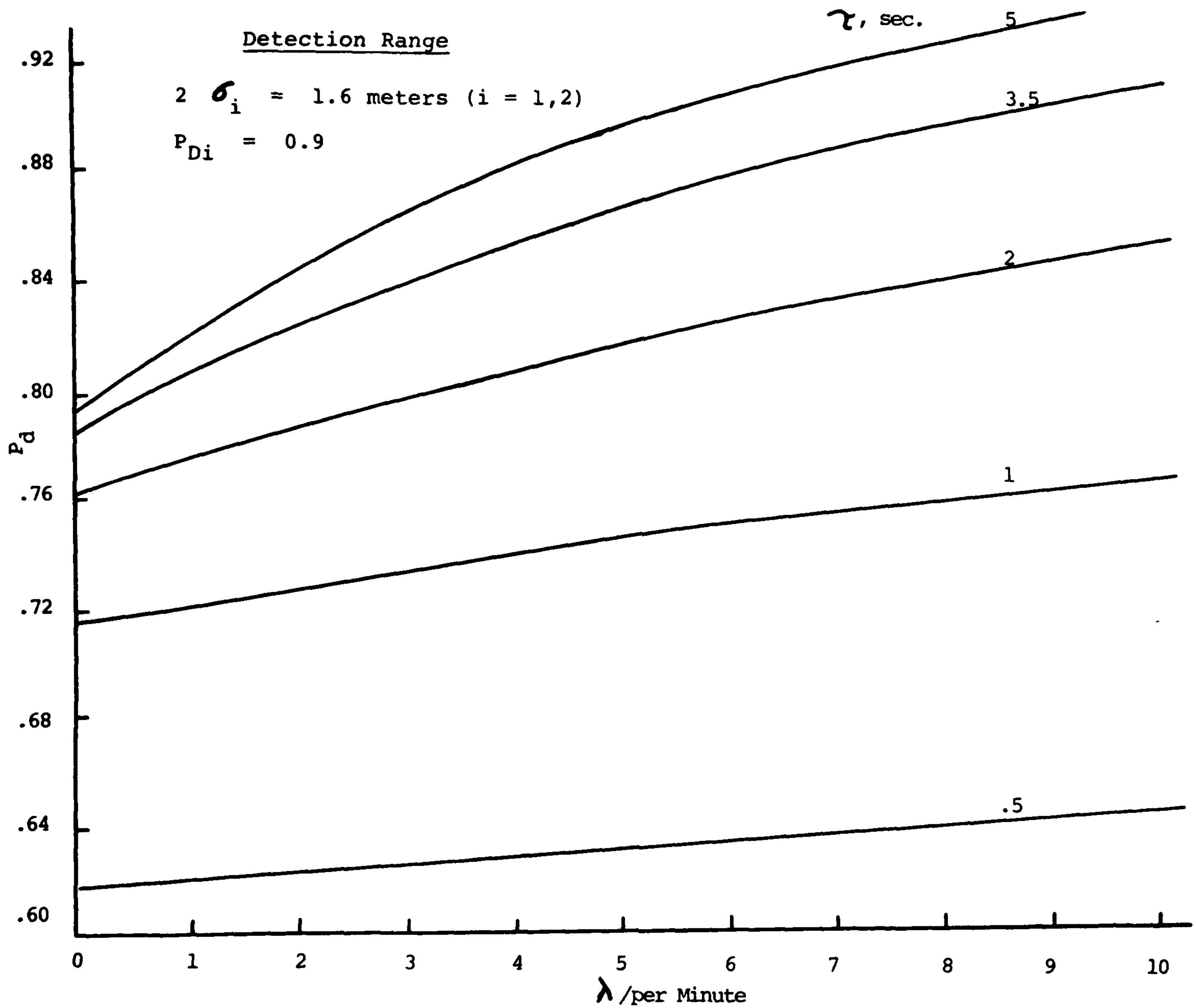


Figure 7c. PROBABILITY OF DETECTION FOR SENSOR SYSTEM ($2\sigma_i = 1.6$m)

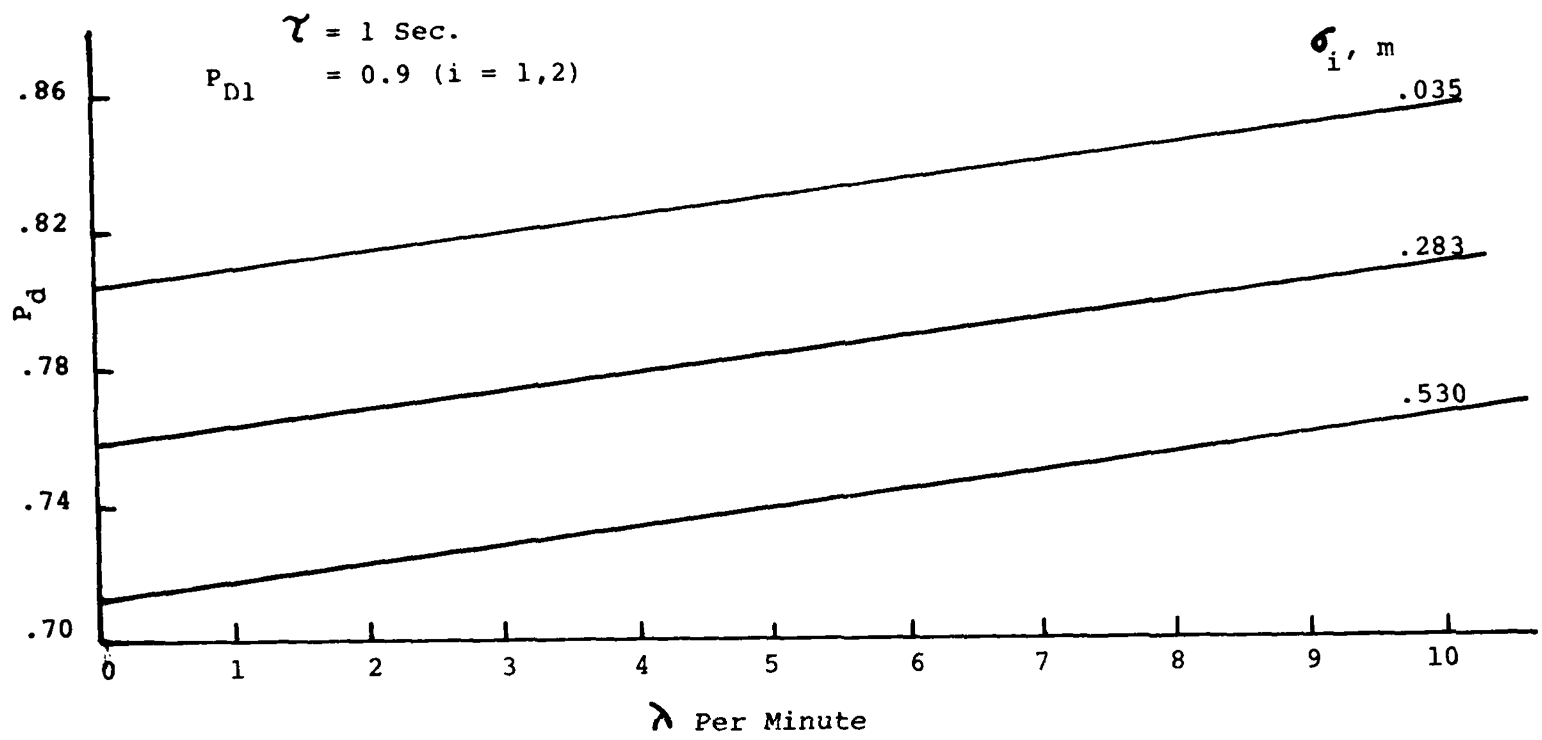


Figure 8a. PROBABILITY OF DETECTION FOR SENSOR SYSTEM (τ = 1 Sec.)

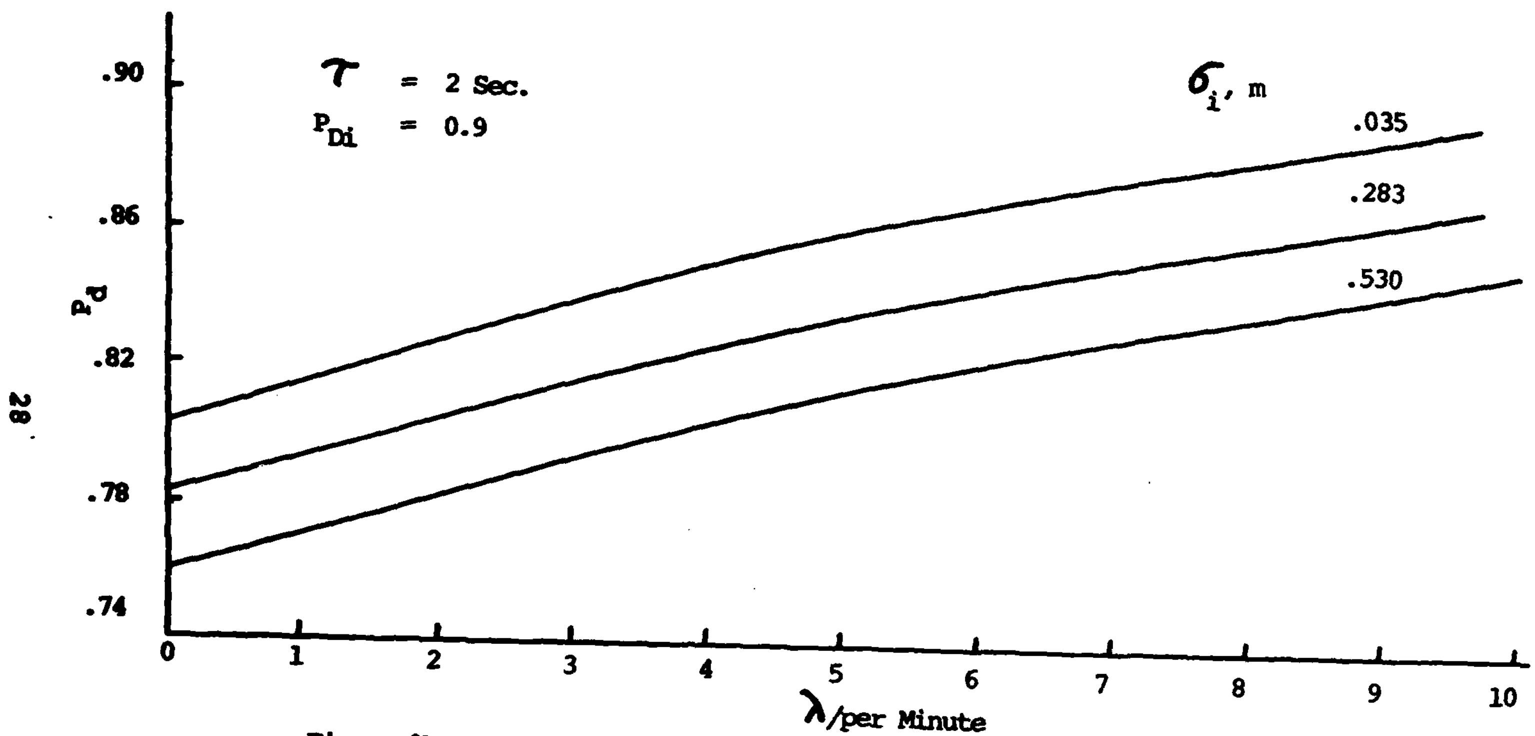


Figure 8b. PROBABILITY OF DETECTION FOR SENSOR SYSTEM (τ = 2 Sec.)

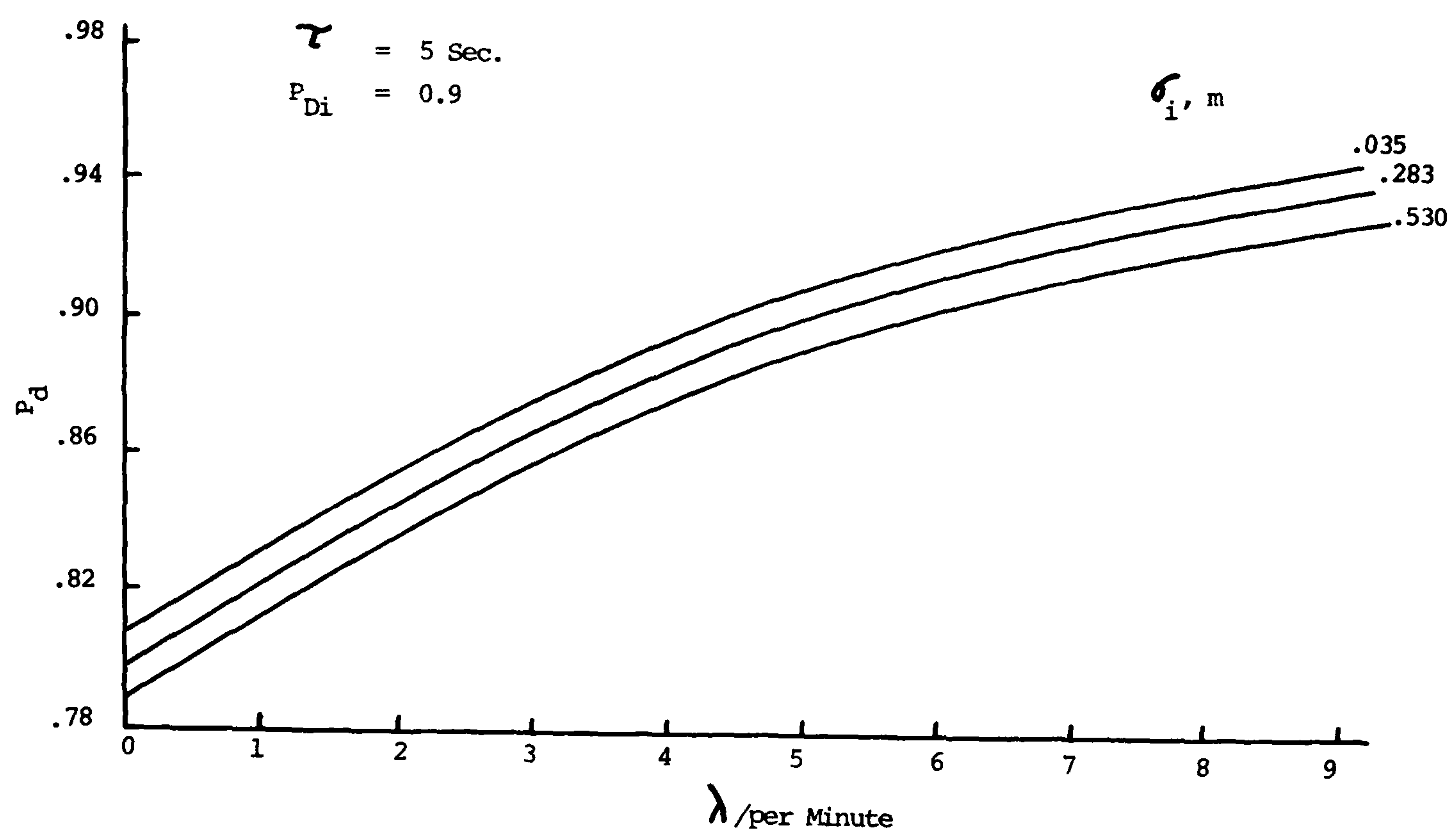


Figure 8c. PROBABILITY OF DETECTION FOR SENSOR SYSTEM (τ = 5 sec.)

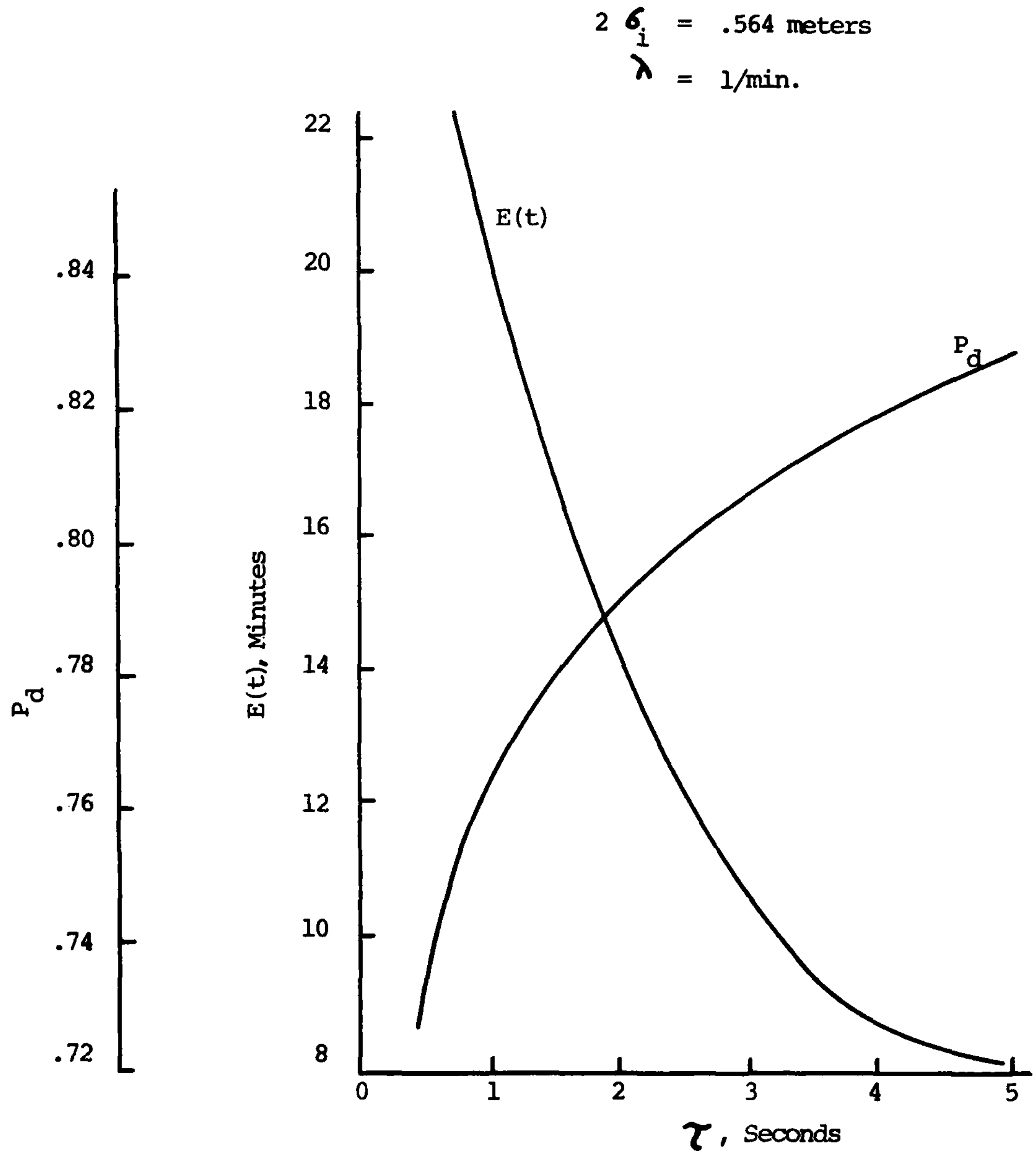


Figure 9a. ASSESSMENT TIME VS. DETECTION DEGRADATION TRADE OFF
(λ = 1/Min.)

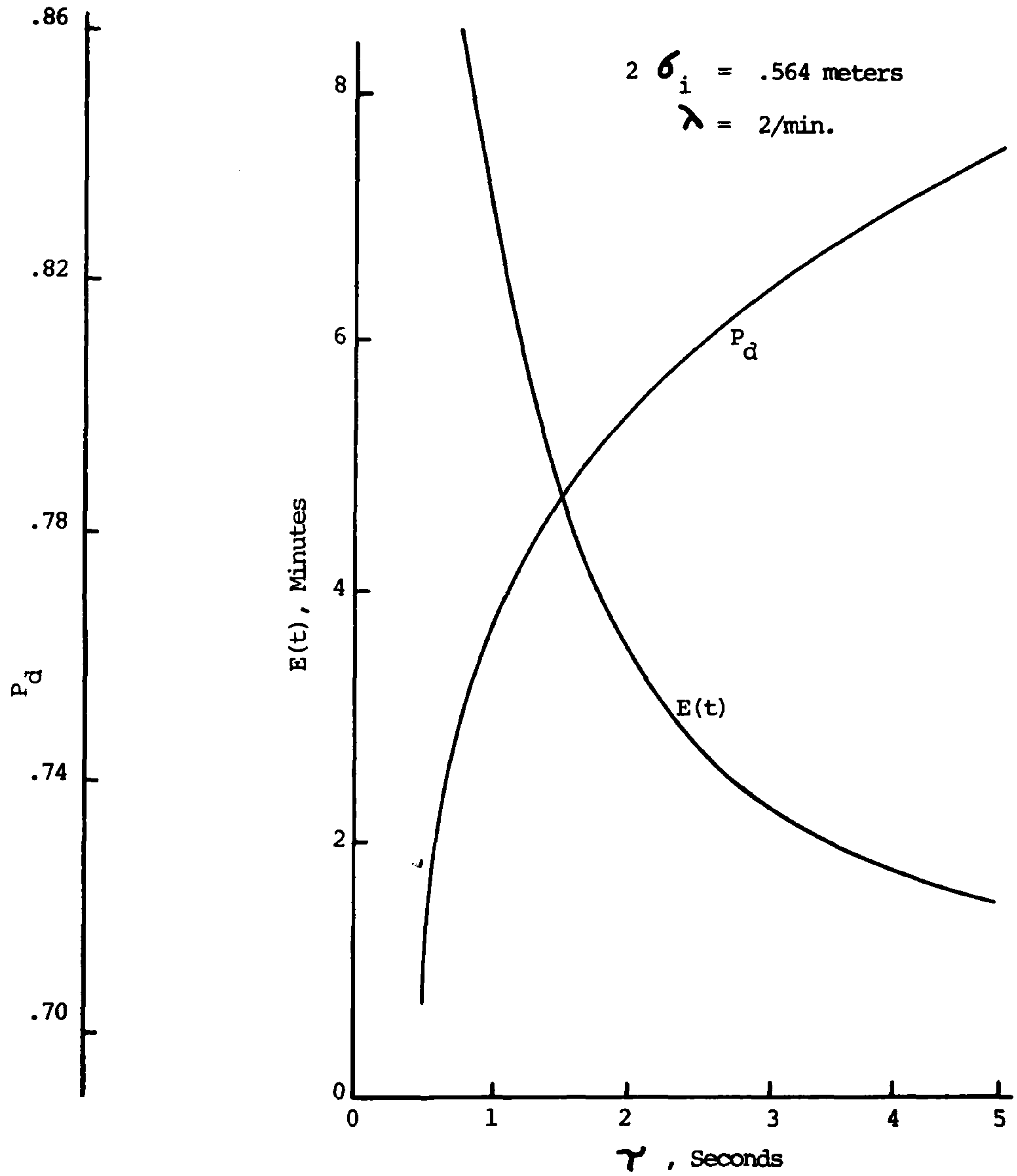


Figure 9b. ASSESSMENT TIME VS. DETECTION DEGRADATION TRADE-OFF
(λ = 2/Min.)

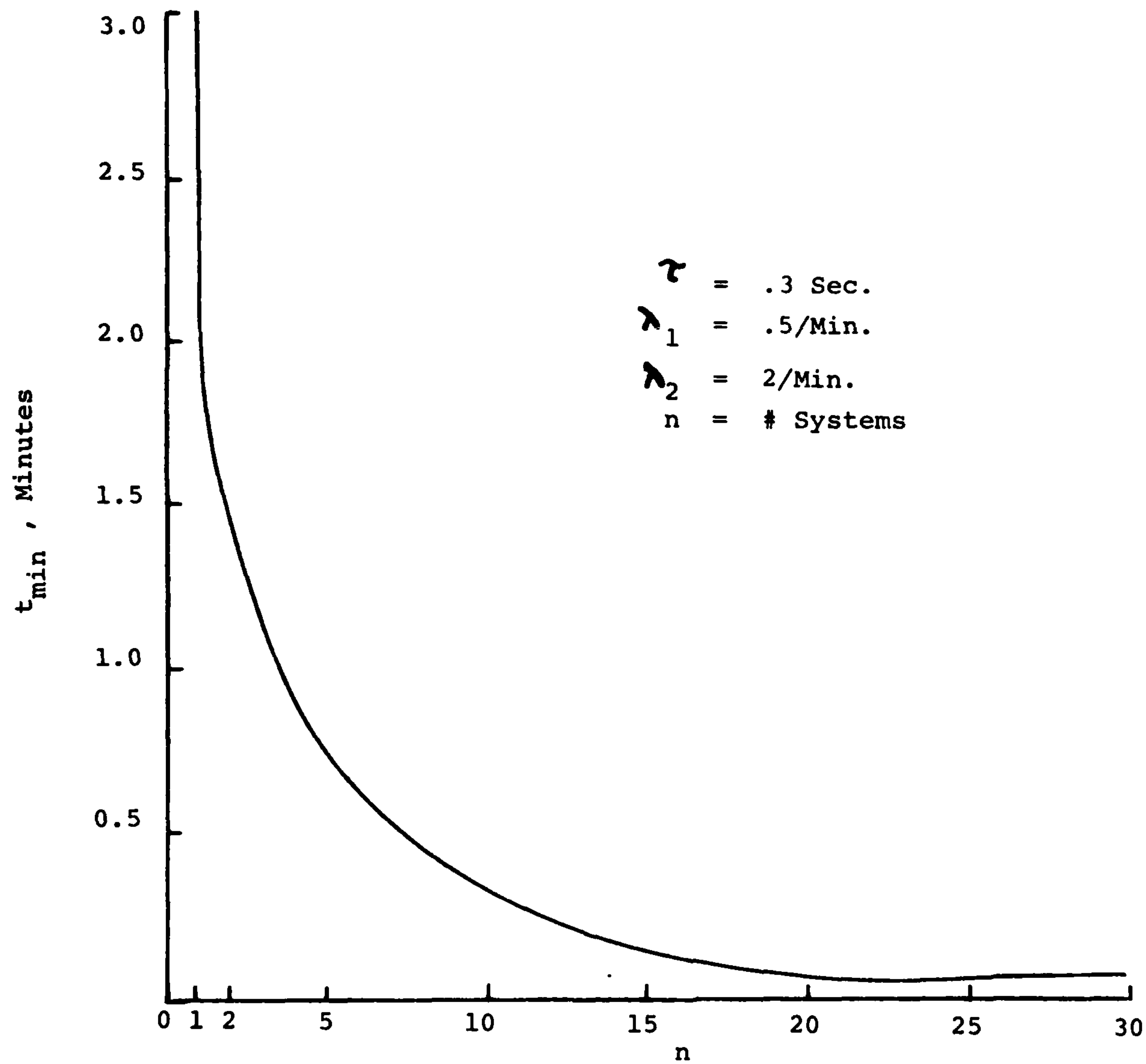


Figure 10. RELATIONSHIP OF t_{min} and n

common τ-dependence. Figure 11 shows plots of P_d versus t_{min} for n = 1,2 and 10. From the plot for n = 10, it is seen that when t_{min} = 0.5 min the system P_d is 0.9. For this case, the average time between alarms for the combined system is approximately 10 minutes. If S_1 (λ_1 = 0.5) were used alone in each of the ten zones, the average time between alarms would be 12 seconds. Moreover, t_{min} would be equal to 0.6 seconds as against the 0.5 min. when the combined system is used.

SUMMARY AND CONCLUSIONS

The numerical example presented above indicates that substantial improvement in the false alarm behavior of peripheral detectors may be achieved by combining two such detectors in an AND-logic mode. It should be emphasized that the detection model used in that example permitted only one detection per sensor during an intrusion. Realistic sensors will not have this limitation and therefore, the predicted detection probability performance is conservative.

To definitively assess the value of a combined system the characteristics of actual sensors must be embodied in the detection model. This includes the false alarm behavior (which may or may not be Poisson) as well as communication delays and lockout times. When all of these factors are included, the best approach to an evaluation of the combined system is probably a Monte Carlo simulation. This will be particularly true if detailed examination of actual false alarm data reveals that the statistics of these data do not obey a law that is amenable to analytic manipulation.

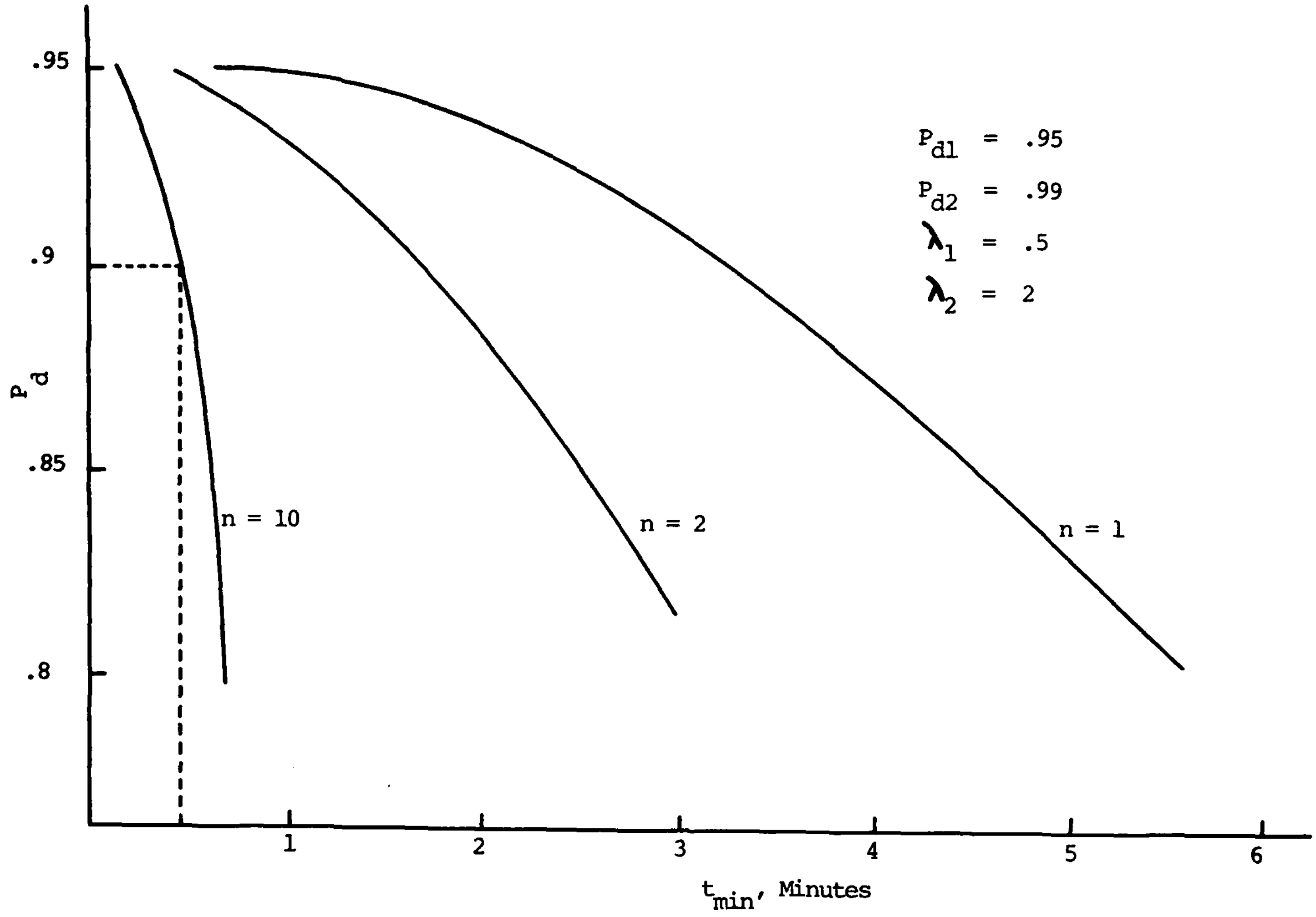


Figure 11. RELATIONSHIP OF P_d and t_{min}

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